HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1981

數學(課程一) 試卷一 MATHEMATICS (SYLLABUS 1)
PAPER I

二小時完整

上午八時三十分至上午十時三十分

本試卷必須用英文作客

Two hours 8.30 a.m.—10.30 a.m.

This paper must be answered in English

Attempt ALL questions in Section A and any FIVE questions in Section B. Full marks will not be given unless the method of solution is shown.

FORMULAS FOR REFERENCE

SPHERE	Surface area	*	4#12
	Volume	100	4 # P B
CYLINDER	Area of curved surface		2 nrh
	Volume	=	22° h
CONE	Area of curved surface	-	art
	Volume	-	$\frac{1}{3}\pi r^{\pm}h$
PRISM	Volume	-	baso area X height
PYRAMID	Volume	-	$\frac{1}{2}$ X best free X heigh

SECTION A Aparter ALL questions in this section.

There is no need to start each question in this section on a fresh page.

1. The expecities of two spherical tanks are in the ratio 27:64. If 72 kg of paint is required to paint the outer surface of the smaller tank, then how many kilograms of paint would be required to paint the outer surface of the bigger tank?
(5 marks)

2. Find a quadratic equation whose roots are

2 + 3t and 2 - 3t where $t = \sqrt{-1}$;

Express your answer in the form $x^1 + bx + c = 0$ where b and c are real numbers.

(5 marks)

 There are 40 students in a class, including students A and B. If two students are to be chosen at rendom as class representatives, find the probability that both A and B are chosen.

(5 merice)

4. Solve cos(200° + 0) = sin 120° where 0° < 0 < 180°.

(6 marks)

5. Solve $4^{x} = 10 - 4^{x+1}$

(6 marks)

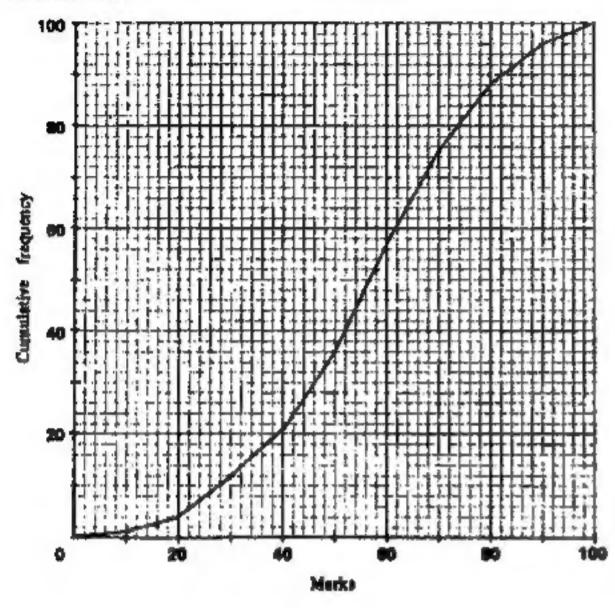


Figure 1

- (a) If 75% of the students pass the test, what is the pass mark, correct to the nearest integer?
- (b) If the pass mark were 40, how many students would pass the test?
- (c) Find the inter-quartile range.

(6 marks)

- 7. In a clear of 42 students, 28 have been to Ocean Park and 34 have been to the Space Museum
 - (a) Find the least number of students who have been to both Ocean Park and the Space Museum.
 - (b) If 7 of the 42 students have never been to Ocean Park or the Space Museum, find the number of students who have been to both places.

(6 marka)

Candidate Number Centre Number Seat Number on this page

SECTION I

Answer FIVE questions in this section, Each question carries 12 marks.

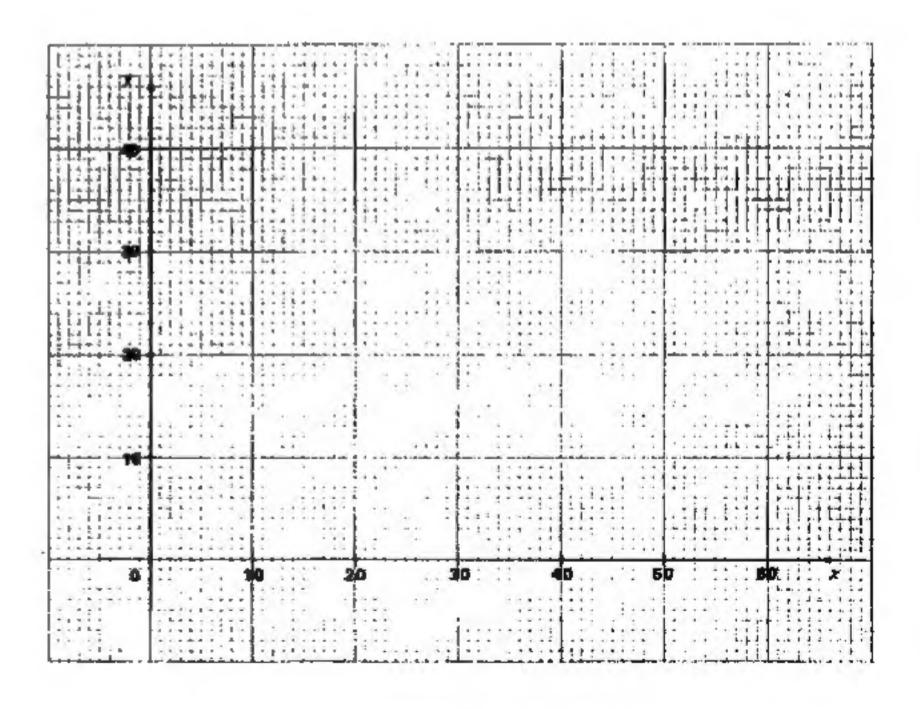
8. If you attempt this question, fill in the details in the first three boxes above and tie this sheet into your unever book.

An association plans to build a hostel with x single rooms and y double rooms satisfying the following conditions:

- (1) The hostel will eccommodate at least 48 persons.
- (2) Each single room will occupy an area of 10 m³, each double room will occupy an area of 15 m³ and the total available floor area for the rooms is 450 m³
- (3) The number of double rooms should not exceed the number of single rooms.

If the profits on a single room and a double room are \$300 and \$400 per month respectively, find graphically the values of x and y so that the total profit will be a maximum.

(12 marks)



(12 marks)

10.

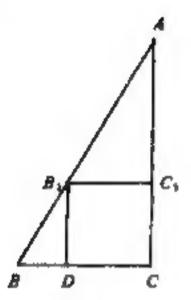


Figure 2(a)

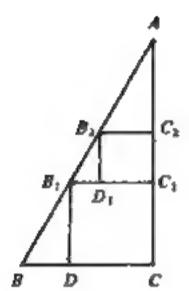


Figure 2(b)

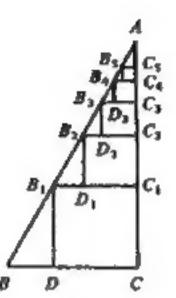


Figure 2(c)

In Figure 2(a), B_1C_1CD is a square inscribed in the right-angled triangle ABC. $LC = 90^\circ$, BC = a, AC = 2a, $B_1C_1 = b$.

(a) Express b in terms of a.

(3 marks)

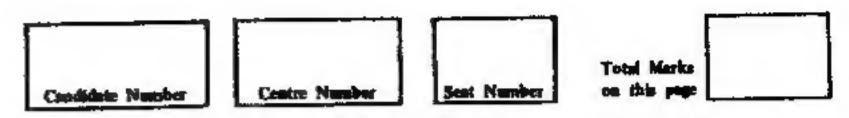
- (b) $B_2 C_2 C_1 D_1$ is a square inscribed in $\Delta A B_1 C_2$ (see Figure 2(b)).
 - (i) Express $B_2 C_2$ in terms of b:
 - (ii) Hence express B2 C2 in terms of a.

(2 marks)

- (c) If squares $B_3 C_2 C_2 D_2$, $B_4 C_4 C_3 D_3$, $B_3 C_2 C_4 D_4$, ... are drawn successively as indicated in Figure 2(c),
 - (i) write down the length of B, C, in terms of a,
 - find, in terms of a, the man of the areas of the infinitely many squares drawn in this way.

(7 marks)





11. If you attempt this question, fill in the details in the first three boxes above and tie this sheet into your nature book.

A place of wice 20 cm long is bent into a rectangle. Let one side of the rectangle be x cm long and the area be y cm².

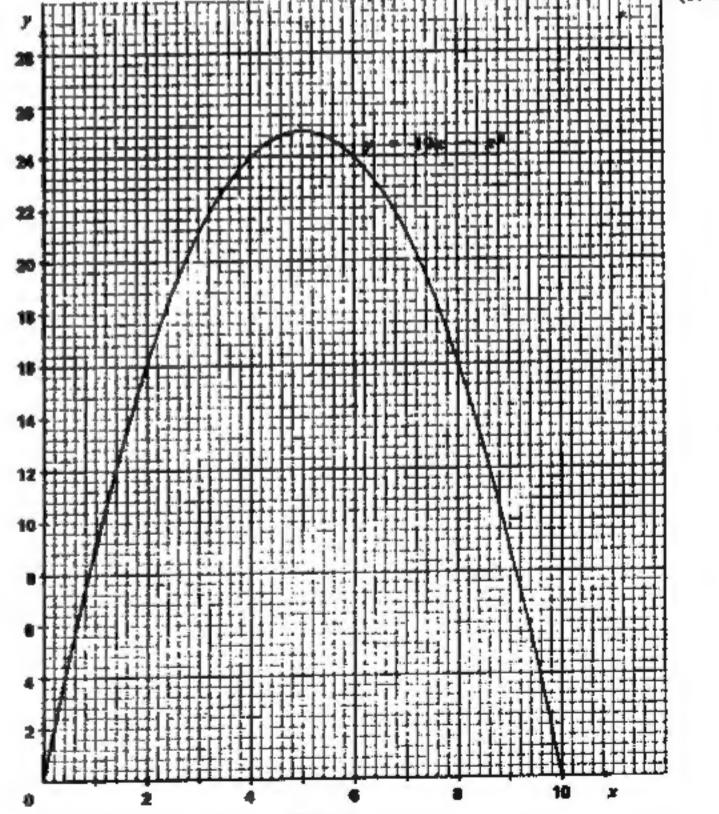
(a) Show that $y = 10x - x^3$

(2 marks)

- (b) Figure 3 shows the graph of $y = 10x x^2$ for $0 \le x \le 10$.

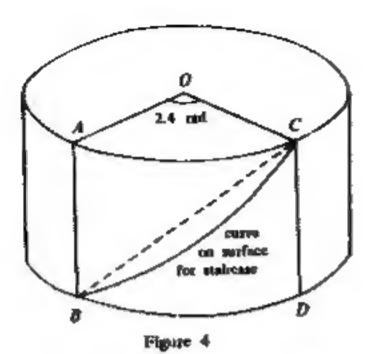
 Using the graph, find (i) the value of y correct to 1 decimal place, when x = 3.4.
 - (ii) the values of x correct to i decimal place, when the area of the rectangle is 12 cm².
 - (iii) the greatest area of the rectangle,
 - (iv) the value of x for which y is three times x, by drawing a suitable line on the graph.

(10 marks)



B1-CE-MATHS (BYL 1) 1-8

Figure 3



(a) louide the cylinder, a straight pipe runs from B to C. Calculate the length of the pipe BC correct to 3 significant figures. (5 marks)

(b) Calculate the area of the curved surface ABDC bounded by the minor area AC, BD and the lines AB, CD. (3 marks)

(c) A staircase from B to C is built along the shortest curve on the curved surface ABDC. Find the length of the curve. (4 marks)

13.

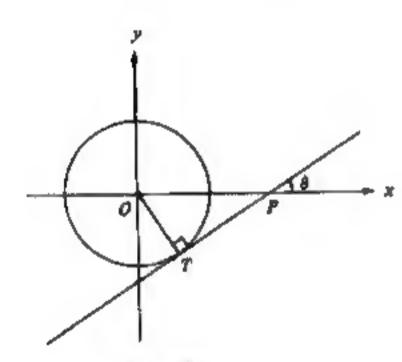


Figure 5(a)

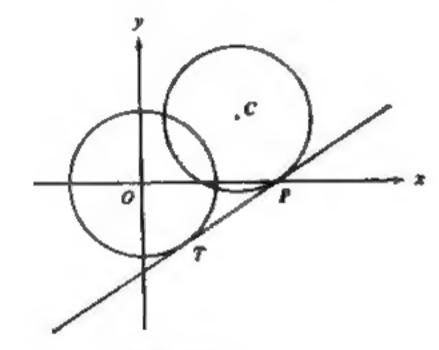


Figure 5(b)

Figure 5(a) shows a circle of radius 15 with centre at the origin O. The line TP, of slope $\frac{3}{4}$ (\approx tan θ), touches the circle at T and cuts the x-axis at P.

(a) Find the equation of the circle.

(1 mark)

(b) Calculate the length of OP.

(3 madks)

(c) Find the equation of the line TF.

(2 marks)

Another circle, with centre C and radius 15, is drawn to touch TP at P (see Figure 5(b)).

(d) Find the equation of the line OC.

(1 mark)

(e) Find the equation of the circle with centre C.

(5 marks)



\$1-CE-MATHS (BYL 1) 1-7

Condidate Number

Centre Number

Seat Number

Total Marks on this page

14. If you attempt this question, fill in the details in the first three boxes above and tie this pheer into your answer book.

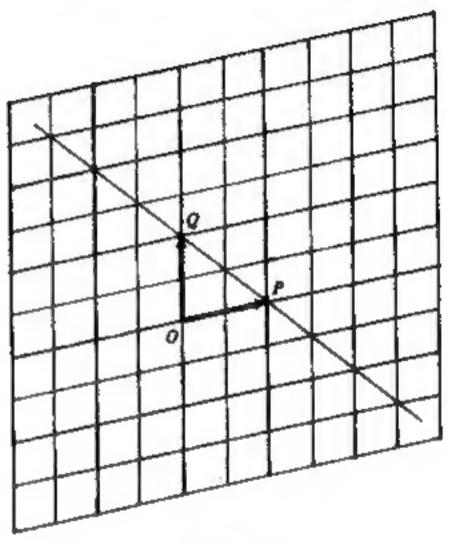


Figure 6

In Figure 6, $\overrightarrow{OP} = \overrightarrow{p}$ and $\overrightarrow{OQ} = \overrightarrow{q}$.

(a) If $\overrightarrow{OA} = \frac{1}{2} \overrightarrow{p} + \frac{1}{2} \overrightarrow{q}$ and $\overrightarrow{OB} = \frac{3}{2} \overrightarrow{p} - \frac{1}{2} \overrightarrow{q}$, draw the vectors \overrightarrow{OA} and \overrightarrow{OB} onto Figure 6.

(2 marks)

(b) T is a point on PQ produced such that $\overrightarrow{QT} = \frac{1}{2} \overrightarrow{PQ}$. Express \overrightarrow{QT} in seems of \overrightarrow{p} and \overrightarrow{q} .

(3 marks)

(c) Let $\vec{p} = 6\vec{7} + 2\vec{f}$ and $\vec{q} = 5\vec{f}$ where \vec{f} and \vec{f} are perpendicular unit vectors. R is a point such that

OR = rp + (1 -r) 4.

(i) Express the dot product $\overrightarrow{PQ} \cdot \overrightarrow{OR}$ in terms of r

(ii) If $\overrightarrow{PQ} + \overrightarrow{OR}$, find the value of r.

(7 marks)

END OF PAPER

81-CE-MATHO (8YL 1) 1-6

香港考試展

HONG KONG EXAMINATIONS AUTHORITY

一九八一年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1981

數學(課程二) 試卷一

二小時完整

上华八時三十分至上午十時三十分

本試卷必須用英文作答

MATHEMATICS (SYLLABUS 2) PAPER I

Two hours

8.30 a.m.—10.30 a.m.

This paper must be answered in English

Attempt ALL questions is Section A and any FIVE questions in Section B. Full marks will not be given unless the method of solution is shown.

FORMULAS FOR REFERENCE

SPHERE	Surface area		4173
	Volume	-	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	-	2 meh
	Volume	•	wr ³ h
CONE	Area of curved surface		nri
	Volume	-	$\frac{1}{3}\pi e^{2}h$
PRISM	Volume	¢	base sees X height
PYRAMID	Volume	•	1 × hase area × heigh

SECTION A Answer ALL questions in this section.

There is no need to start each question in this section on a fresh page.

Geometry theorems used not be referred to when used.

1. The expecities of two spherical tanks are in the ratio 27:64. If 72 kg of paint is required to paint the outer surface of the smaller tank, then how many kilograms of paint would be required to paint the outer surface of the bigger tank?
(5 marks)

2. If $x = (a + \delta y^2)^{\frac{1}{3}}$, express y in terms of a, b and x. (5 marks)

3. Let f(x) = (x + 2)(x - 3) + 3.

When f(x) is divided by (x - k), the remainder in k. Find k. (5 marks)

4. Suive cos(200° + #) = skn 120° where 0° < # < 180°.
(6 marks)

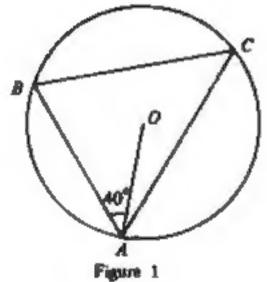
5. Factorize
$$(1 + x)^4 - (1 - x^2)^2$$
,

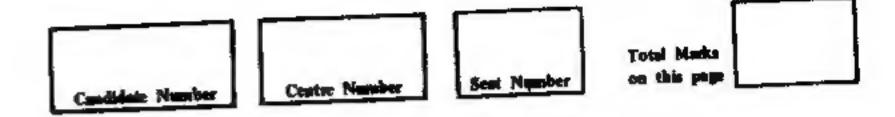
(6 marks)

Solve 4" = 10 - 4" +1 .

(6 marks)

In Figure 1, O is the centre of circle ABC. $\angle OAB = 40^{\circ}$. Calculate L BCA . (6 marks)





SECTION B

Asswer FIVE questions in this section, Each question carries 12 marks,

8. If you attempt this question, 500 in the details in the first three beates show and tie this short into your arrest book.

An association plans to build a hostel with x single rooms and y double rooms satisfying the following conditions:

- (1) The hostel will accommodate at least 45 persons.
- (2) Each single room will occupy an area of 10 m², each double room will occupy an area of $15~{\rm m}^3$ and the total available floor area for the rooms is $450~{\rm m}^3$.
- (3) The number of double rooms should not exceed the number of single rooms.

If the profits on a single room and a double soom are \$300 and \$400 per month respectively, find graphically the values of x and y so that the total profit will be a maximum.

(12 marks)

					-11.000	larger to a ding house	and the second property and
			1.0		street 1 am I to		
10 was -	244		- 1	11 11	14 14 1	doct - process to	*
	1.00	. 144	1 2 4	en			
	1	4. 4.		4		trans house	
314 4 4 4			485		b - i		
Special Control of	*******	4	- 0 0				
	11111		7		and the second	desire days with the state of the	
						*** * * * * * * * * * * * * * * * * * *	
	Sec. 21 1		1	- 1			New York & p. ft.
many I -						the State of the S	and or second a second
- 0	2	444		waller!			the pull is not the first
To a before a							
on I was made a - a		1		4 4 4 1 4			at any time had been
and a second						4 2 3 4 5	4 - 1 1 - 175
I had a			4		44. 99. 4	4 4 5 161	to the history of
Control & a Control	414					4	2 1 4 4 4
		*****	SLEEDE A	1	15.5	A PARALLAND	a Section
west which a war v	**		***	4 4 7 7 7			
A		-					31711111
	1 1 1	4 44			mat 7 4 9		The second second
110 4 4 4	interest I			tild a f	and the same		
APCA TE		********	200		and salam		arran aded
			* * * * * * * * * * * * * * * * * * * *		Laker god a	2 T.	
1.4.3	1::	C 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4		12		7	
		A 44 A 44 A 44	2 22 2 2 2 4 4			3-	the Bottom of Lake
			1111 510			1	4 1 444 1 7 7 1
Salle at 1		THE REAL PROPERTY.		fitation .			
Tel control		**		Acres de la constitución de la c		A	1 1 1 1 1 2 1 4 4 C
The second second			the tree to				. 111771
CARREST STATE	11111			111 11			
without a s t	1111			11.2		1 mm 2 3	
many chieves	1			***		grand f	
		/				11	
7 cm 14 mm							
NAME OF TAXABLE						1	
The state of the						1	
				1 14		1	
	145			1	1		
	1 1 3 3 4 4 4 4 5			7 1 2 200	14 1 1 1 1 4 4 4		
I u u e e e	222340 00						1 1 1 1 1 1 1 1
and become to			4 4	1 ***	444 4	4	1
Land Strain St. S . S	***		4. 7.5	1	1 212 111 1	1	
11544345					4 .	24.44	1
- the same			9		9	*** * ** * * * * * * * * * * * * * * * *	
			4				
11:00	-			7	W	5d	A X .
11 A 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1			20	10.			
	11.						
	. 1			. 41,9191 5			1
	1 3000			A 313131 5 5	1		4
	1			343.57			+ +1 + +
	1			1 mm + 2 4 7 1 1		1 to . See	4-4 4 ·
American Contract	1 kan 3 2 3	0.0001		** * * * * * * * * * * * * * * * * * * *	1	- where one mist	1 44 ma 4 44 4 4
Canada tet 1 c				1.			

(12 marks)

10.

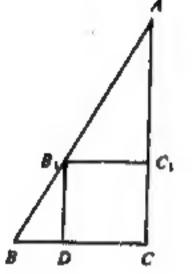


Figure 2(a)

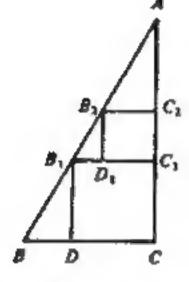
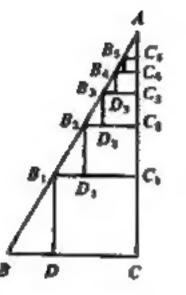


Figure 2(b)



Pigure 2(c)

In Figure 2(a), B_1C_1CD is a square inscribed in the right-angled triangle ABC. $LC=90^o$, BC=a, AC=2a, $B_1C_1=b$.

(s) Express b in terms of a.

(3 marks)

- (b) $B_2 C_2 C_1 D_1$ is a square inscribed in $\triangle AB_1 C_1$ (see Figure 2(b)).
 - (i) Express B2 C2 in terms of b.
 - (ii) Hence express B_1C_1 in terms of a.

(2 murks)

- (c) If equares $B_1C_1C_2D_3$, $B_4C_4C_5D_3$, $B_5C_5C_4D_4$, ... are drawn successively at indicated in Figure 2(c),
 - (i) write down the length of B, C, in terms of a .
 - (ii) find, in terms of s , the sum of the areas of the infinitely many aquares drawn in this way.

(7 marks)



П.

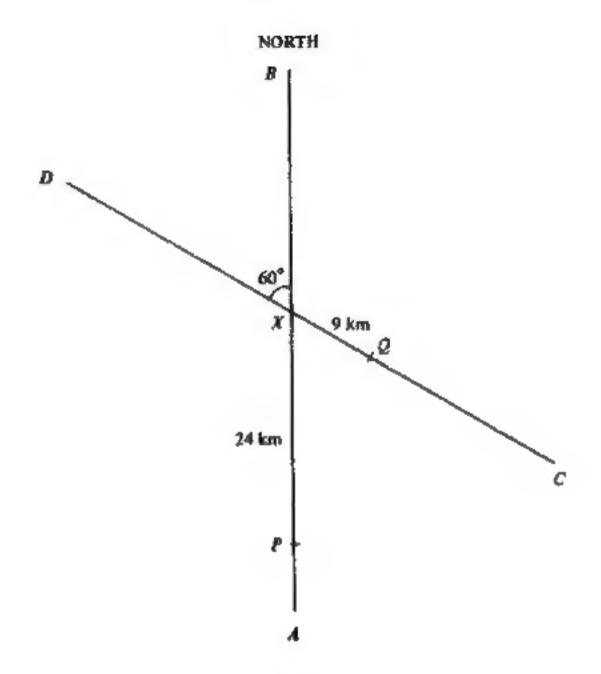


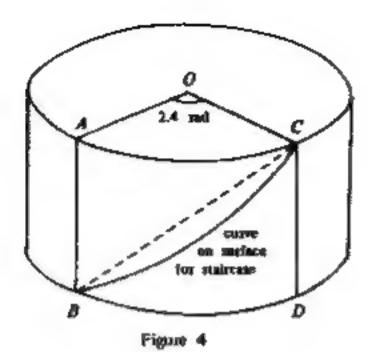
Figure 3

AB and CD are two straight roads intersecting at X. AB cans North and makes an angle of 60° with CD. At noon, two people P and Q are respectively $24 \, \mathrm{km}$ and $9 \, \mathrm{km}$ from X as shown in Figure 3. P walks at a speed of $4.5 \, \mathrm{km/h}$ towards B and Q walks at a speed of $6 \, \mathrm{km/h}$ towards D.

(a) Calculate the distance between P and Q at noon. (4 marks)

(b) What are the distances of P and Q from X at 4 p.m.? (2 marks)

(c) Calculate the hearing of Q from P at 4 p.m. to the nearest degree. (6 marks)



- Inside the cylinder, a straight pipe runs from B to C. Calculate the length of the pipe BC correct to 3 significant figures.
- Calculate the area of the curved surface ABDC bounded by the minor eres AC, BD and the lines AB, CD.
- A staircase from B to C is built along the shortest curve on the curved surface ABDC. Find the length of the curve. (4 marks)
- In Figure 5, circles PMQ and QNR touch each other at Q. QT is a common tangent. PQR is a straight line. TP and TR out the circles at M and N respectively.

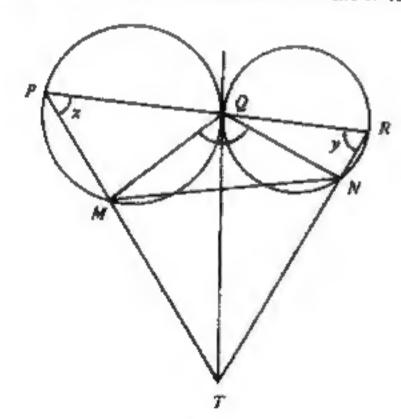


Figure 5

- If $\angle P = x$ and $\angle R = y$, express $\angle MQN$ in terms of x and y. (2 marks)
- Prove that Q, M, T and N are concyclic. (3 murles)
- Prove that P, M, N and R are concyclic. (4 marks)
- There are several pairs of similar triangles in the figure. Name any two pairs (no proof is required).

(3 merks)

14,

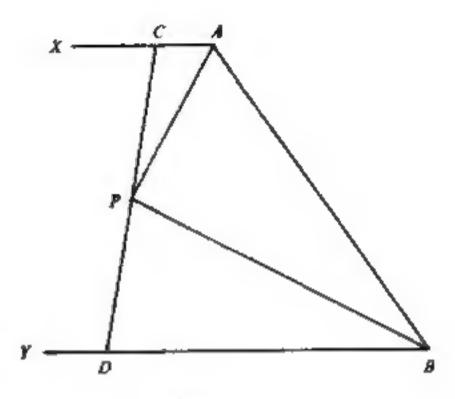


Figure 6

In Figure 6 , AX PBY . AP and BP bisect LXAB and LYBA respectively, and they meet at P. A straight line passing through P meets AX and BY at C and D respectively.

Prove that (a) $\angle APB = 90^{\circ}$,

(4 marks)

(b) CP = DP,

(5 marks)

(c) AC + BD = AB.

(3 marks)

END OF PAPER

香港考試局

HONG KONG EXAMINATIONS AUTHORITY

一九八一年香港中學會考 HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1981

數學(課程三) 試卷一

二小時常卷 上午八時三十分至上午十時三十分 本試卷必須用英文作客

MATHEMATICS (SYLLABUS 3) PAPER I

Two hours
8.30 a.m.—10.30 a.m.
This paper must be answered in English

Attempt ALL questions in Section A and any FIVE questions in Section B. Full marks will not be given unless the method of solution is shown.

FORMULAS FOR REFERENCE

SPHERE	Surface area	-	4 mr 2
	Volume	*	4/3 #F3
CYLINDER	Area of curred surface	•	2 # ch
	Volume	*	wr2h
CONE	Area of curved surface		wri
	Volume	12	$\frac{1}{3}\pi r^2 A$
PRISM	Votume:	•	base area X height
PYRAMID	Volume		$\frac{1}{3}$ X base area X height

SECTION A Answer All questions in this section.

There is no need to start each question in this section on a fresh page.

The capacities of two apherical tanks are in the ratio 27:64. If 72 kg of paint is required to paint the outer surface of the smaller tank, then how many kilograms of paint would be required to paint the outer surface of the bigger tank?

(5 marks)

2. Let f(x) = (x + 2)(x - 3) + 3. When f(x) is divided by (x - k), the semainder is k. Find k. (5 marks)

3. There are 40 students in a class, including students A and B. If two students are to be chosen at random as class representatives, find the probability that both A and B are chosen.
(5 marks)

4. Solve $cos(200^{\circ} + \theta) = sin 120^{\circ}$ where $0^{\circ} \le \theta \le 180^{\circ}$.

(6 marks)

BI-CE-MATHS (BYL 3) 1-2

5. Factorize $(1 + \pi)^4 - (1 - \pi^2)^2$.

(6 marks)

- 6. The heights of 1000 students form a symmetrical distribution with a mean of 1.70 m and a standard deviation of 0.02 m. If 67% of the students its within one standard deviation of the mean and 97% lie within two standard deviations of the mean, find
 - (a) the number of students who are shorter than 1.74 m .
 - (b) the number of students whose heights lie between 1.68 m and 1.74 m .

(6 media)

- 7. The parabola $y^3 = 4 a x$ passes through the points A(1, 4) and B(16, -16). A point P divides AB internally such that AP: PB = 1:4.
 - (a) Find the coordinates of P.
 - (b) Show that P is the focus of the given parabols.

(6 marks)

Candidate Number Centre Number Sest Number on this page

SECTION B

Answer FIVE questions in this section, Each question cervies 12 marks.

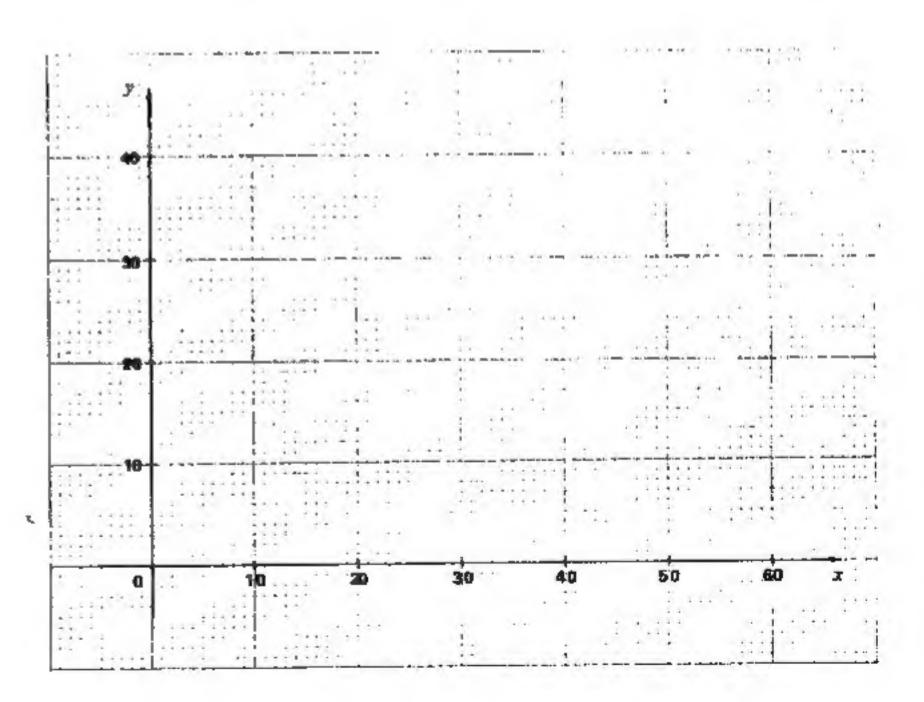
8. If you attempt this question, fill in the details in the first three boxes above and tie this sheet into your mover book.

An association plane to build a hostel with x single rooms and y double rooms satisfying the following conditions:

- (1) The hostel will accommodate at least 48 persons.
- (2) Each single room will occupy an area of 10 m^2 , each double room will occupy an area of 15 m^2 and the total available floor area for the rooms is 450 m^2 .
- (3) The number of double rooms should not exceed the number of single rooms.

If the profits on a single room and a double room are \$300 and \$400 per month respectively, find graphically the values of x and y so that the total profit will be a maximum.

(12 marks)



(12 marks)

10.

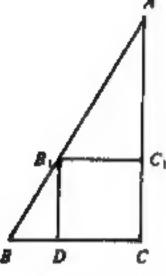


Figure 1(a)

<u>_</u>

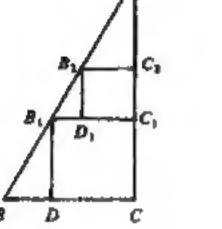


Figure 1(b)

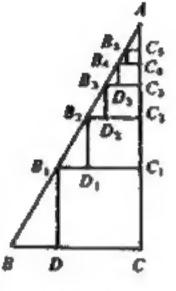


Figure 1(c)

In Figure 1(a), B_1C_1CD is a square inscribed in the right-angled triangle ABC. $LC=90^\circ$, BC=a, AC=2a, $B_1C_1=b$.

(a) Express b in terms of a.

(3 marks)

- (b) $B_2 C_1 C_1 D_1$ is a square inscribed in $\triangle AB_1 C_2$ (see Figure 1(b)).
 - (i) Express B₂C₃ in terms of b.
 - (ii) Hence express $\theta_1 C_2$ in terms of a.

(2 merku)

- (c) If squares $B_3 C_3 C_4 D_4$, $B_4 C_4 C_3 D_3$, $B_3 C_4 C_4 D_4$, ... are drawn successively as indicated in Figure 1(c),
 - (i) write down the length of 8, C; to terms of a .
 - (ii) find, in terms of a, the sum of the areas of the infinitely many squares drawn in this way.

(7 marks)

11.

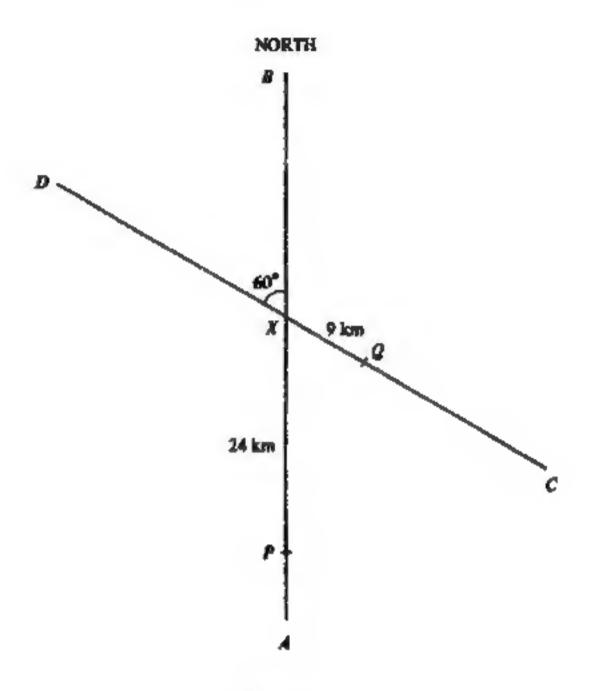


Figure 2

AB and CD are two straight roads intersecting at X. AB runs North and makes an angle of 60° with CD. At noon, two people P and Q are respectively 24 km and 9 km from X as shown in Figure 2. P walks at a speed of 4.5 km/h towards B and Q walks at a speed of 6 km/h towards D.

(a) Calculate the distance between P and Q at noon. (4 marks)

(b) What are the distances of P and Q from X at 4 p.m.? (2 marks)

c) Calculate the bearing of Q from P at 4 p.m. to the nearest degree. (6 marks)

high and 10 metres in radius used for storing coal-gas. AB and CD are two vertical lines on the curved surface of the cylinder. The arc AC subtends an angle of 2.4 radians at the point O, which is the centre of the top of the cylinder.

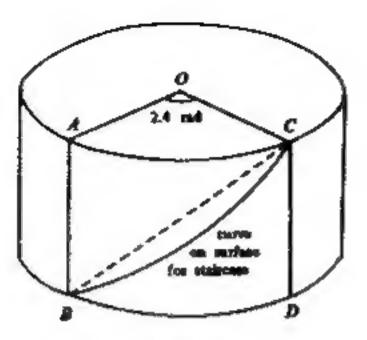


Figure 3

- (a) Inside the cylinder, a straight pipe runs from # to C. Calculate the length of the pipe #C correct to 3 significant figures. (5 snarks)
- (b) Calculate the area of the curved surface ABDC bounded by the minor area AC, BD and the lines AB, CD. (3 marks)
- (c) A staircase from B to C is built along the shortest curve on the curved surface ABDC. Find the length of the curve. (4 merbs)

13.

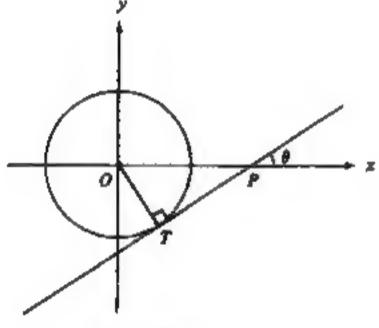


Figure 4(a)

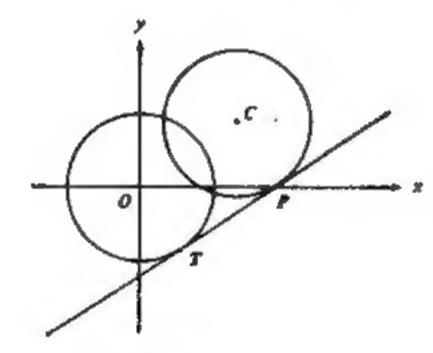


Figure 4(b)

Figure 4(a) shows a circle of radius 15 with centre at the origin O. The line TP, of slope $\frac{3}{4}$ (= ten θ), touches the circle at T and cuts the x-axis at P.

(a) Find the equation of the circle.

(1 mark)

(b) Calculate the length of OP.

(3 marks)

(c) Find the equation of the line 77.

(2 marks)

Another circle, with centre C and radius 15, is drawn to touch TP at P (see Figure 4(b)).

(d) Find the equation of the line OC.

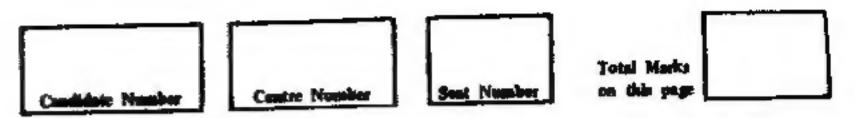
(1 mmk)

(c) Find the equation of the circle with centre C.

(5 merks)



BI-CE-MATHS (SYL 3) (-7



14. If you attempt this question, fill in the details in the first three boxes above and tie this sheet into your sawwer book.

The relationship between the height y of a flying object and time x is given by

$$y = x^3 + ax^2 + bx$$

where y is in kilometres above sen-level and x is the number of hours after 12:00 noon. Figure 5 shows the graph of $y = x^3 + ax^2 + bx$.

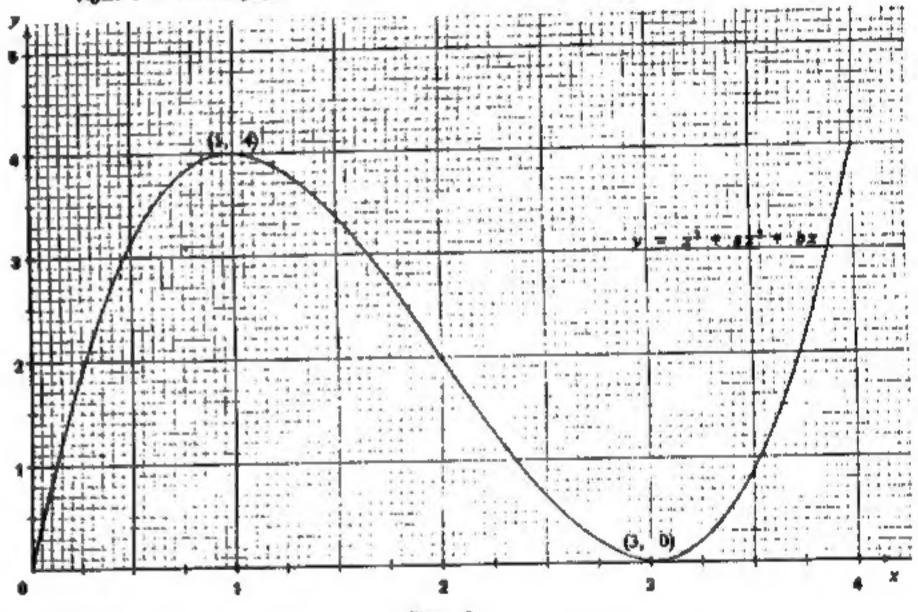


Figure 5

- (a) Using Figure S.
 - (1) find the values of a and b .
 - ii) write down the time interval in which the flying object is descending.

(4 marks)

 $\alpha \alpha$

- (b) At 1:00 p.m., a balloon rises vertically from sea-level with a constant speed of 4 km/h.
 - (i) Add a straight line to Figure S to show the relationship between the height of the balloon and time x
 - (ii) Hence, write down the value of x to 2 significant figures, for which the balloon and the flying object are at the same height. (3 marks)
- (c) Use the method of magnification to find the value of x in (b) (ii) to 3 significant figures.
 (5 marks)

END OF PAPER